Lesson 3: Factoring

The process of factoring polynomials that we study in this lesson "reverses" the process of multiplying polynomials that we studied in the previous lesson. To factor any *sum of terms* means to write as a *product of factors*.

Definition 1. <u>**Finding Greatest Common Factor or (Divisor):**</u> *The term ax^m is the greatest common factor of a polynomial in x if*

1. *a is the <u>greatest</u> integer that divides each of the polynomial coefficients.*

2. *m is the <u>smallest</u> exponent on x found in every term of the polynomial.*

Example 1. *Factor $16x^3 + 24x^2$ and $5x^4 + 20x^2 - 25x$.*
<u>Solutions:</u>

$$16x^3 + 24x^2$$
$$= 8x^2(2x) + 8x^2(4)$$
$$= 8x^2(2x + 4).$$

$$5x^4 + 20x^2 - 25x$$
$$= 5x(x^3) + 5x(4x) + 5x(-5)$$
$$= 5x(x^3 + 4x - 5).$$

Definition 2. <u>**Factoring $Ax^2 + Bx + C$ when $A \neq 1$:**</u>

$$Ax^2 + Bx + C$$
$$= (ax + b)(cx + d)$$
$$= acx^2 + (ad + bc)x + bd.$$

Therefore, we need to find integers $a, b, c,$ and d such that
$$A = ac \qquad C = bd \qquad B = ad + bc.$$

Example 2. *Factor $6x^2 + 17x + 7$ and $4x^2 - 8x - 5$.*
<u>Solutions:</u>
$$6x^2 + 17x + 7 = (6x + \square)(x + \blacksquare)$$

or

$$6x^2 + 17x + 7 = (2x + \diamondsuit)(3x + \blacklozenge)$$

$$6x^2 + 17x + 7$$
$$= (2x + +1)(3x + 7).$$

$$4x^2 - 8x - 5 = (4x + \triangle)(x + \blacktriangle)$$

or

$$4x^2 - 8x - 5 = (2x + \triangledown)(2x + \blacktriangledown)$$

$$4x^2 - 8x - 5$$
$$= (2x + 1)(2x - 5).$$

Definition 3. <u>Factoring by Grouping:</u> *We can sometimes group the terms of a polynomial in such a way that each group has a common factor. This technique is called* ***<u>factor by grouping</u>***.

Example 3. *Factor $6x^3 - 4x - 3x^2 + 2$ and $x^2 + 4x + 4 - y^2$.*
<u>Solutions:</u>

$$6x^3 - 4x - 3x^2 + 2$$
$$= (6x^3 - 3x^2) + (-4x + 2)$$
$$= 3x^2(2x - 1) + (-2)(2x - 1)$$
$$= (2x - 1)(3x^2 - 2).$$

$$x^2 + 4x + 4 - y^2$$
$$= (x^2 + 4x + 4) - y^2$$
$$= (x + 2)^2 - y^2$$
$$= (x + 2 + y)(x + 2 - y).$$

Definition 4. <u>Irreducible Polynomial:</u> *A polynomial with no common factors is called <u>irreducible</u>.*

Example 4. *1. The polynomial $ax + c$ is irreducible if a and c have no common factors.*

2. Every polynomial of the form $x^2 + a^2$ is irreducible.

Example 5. *Factoring:*

$$25x^2 - 49$$
$$= (5x)^2 - 7^2$$
$$= (5x + 7)(5x - 7).$$

Example 6. *Factoring:*

$$x^2 + 10x + 25$$
$$= x^2 + 2.5.x + 5^2$$
$$= (x + 5)^2.$$

Example 7. *Factoring:*

$$8x^3 + 125$$
$$= (2x)^3 + 5^3$$
$$= (2x + 5)((2x)^2 - 2x.5 + 5^2)$$
$$= (2x + 5)(4x^2 - 10x + 25).$$

Example 8. *Factoring:*

$$x^4 - 16$$
$$= (x^2)^2 - 4^2$$
$$= (x^2 + 4)(x^2 - 4)$$
$$= (x^2 + 4)(x + 2)(x - 2).$$